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PLASMA ACCELERATION BY A CLOSED-CIRCUIT HALL CURRENT

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ABSTRACT. The work considers certain physical aspects of a plasma accelerator with a closed circuit Hall current.

INTRODUCTION

A series of works (refs. 1-4) have recently appeared devoted to plasma accelerators with a closed circuit Hall current. An accelerator of this type represents a coaxial element with a radial magnetic field and an axial electric field. The Hall current is closed in the azimuthal direction. In order that the electric current flowing through the accelerator be primarily a Hall current it is necessary that the following condition be satisfied: $\omega\tau \gg 1$, where /294*

$\omega = \frac{eH}{mc}$ is the electron gyrofrequency and τ is the relaxation time of the electron impulse.

The magnetic field H and the axial dimension L of the accelerator are selected in such a way that $\rho_+ \gg L$ (ρ_+ is the Larmor ion radius), and it can be assumed that the magnetic field does not affect the ions. The ions are accelerated by the axial electric field. It is of interest to consider certain physical aspects of such an accelerator.

SECTION 1. THE EQUILIBRIUM OF FORCES IN THE ACCELERATOR

In the stationary case we can write the following equilibrium condition for the electron gas

$$\frac{e}{m} n_e \mathbf{E} + \frac{e}{mc} [\mathbf{j} \times \mathbf{H}] + \frac{1}{m} \nabla p + \frac{\mathbf{j}}{\tau} = 0. \tag{1.1}$$

Here n_e is the concentration of electrons; $E=\nabla\varphi$ is the electric field intensity; $j=n_e v_e$ is the density of electron current; $p=en_e T_e$ is the electron pressure; T_e is the temperature expressed in units of the potential; $e>0$ is the elementary charge.

We shall select the system of coordinates in such a way that the electric field is directed opposite to the x axis while the magnetic field is directed along the z axis and the electron drift takes place in the direction of the y axis.

* Numbers in the margin indicate pagination in original foreign text.

Then we have

$$en_e \frac{d\varphi}{dx} - \frac{dp}{dx} - \frac{eH}{c} j_y \left(1 + \frac{1}{\omega^2\tau^2}\right) = 0, \tag{1.2}$$

$$j_y = \omega\tau j_x, \tag{1.3}$$

$$j_x = b_\perp \left(n_e \frac{d\varphi}{dx} - \frac{1}{e}\frac{dp}{dx}\right), \tag{1.4}$$

where

$$b_\perp = \frac{c\tau}{m} \frac{1}{1+\omega^2\tau^2}. \tag{1.5}$$

From (1.2), by taking into account equations /295

$$\frac{d^2\varphi}{dx^2} = 4\pi e(n_e - n_+), \tag{1.6}$$

$$\frac{dH}{dx} = \frac{4\pi e}{c} j_y \tag{1.7}$$

when $\omega\tau \gg 1$, and assuming that the ions are moving along the electrical field, we find the following expression for the density f of the impulse carried away by the ions:

$$f = p_0 - p_1 + \frac{H_0^2 - H_1^2}{8\pi} - \frac{E_0^2 - E_1^2}{8\pi}. \tag{1.8}$$

In equation (1.8) the subscript "0" designates the values existing at the anode while the subscript 1 designates the values at the output from the accelerator.

In the case when the magnetic field of the Hall current

$$\Delta H = H_0 - H_1 = \frac{4\pi e}{c} \int_{x_1}^{x_0} j_y dx$$

is small compared with the external magnetic field H

$$\Delta H \ll H,$$

it is more convenient to express the quantity f in terms of the Hall current produced by the electric drift.

$$I = \int_{x_1}^{x_2} n_e \frac{c}{H} \frac{d\varphi}{dx} dx. \tag{1.9}$$

Then we can write

$$f = \frac{HI}{c} - \frac{E_0^2}{8\pi}, \tag{1.10}$$

where it is assumed that the electric field at the output vanishes.

We finally have

$$I = j_+\rho_+ + j_{+c}\rho_{+c} + I_0, \tag{1.11}$$

where j_+ is the density of the ion current at the output; ρ_+ is the Larmor ion radius due to the total application of the potential difference φ_0, while j_{+c} and ρ_{+c} are the density of the ion current formed in the z accelerator and the average Larmor radius of the ions respectively.

$$I_0 = \frac{cE_0^2}{8\pi H}. \tag{1.12}$$

SECTION 2. POTENTIAL DISTRIBUTION

When the pressure of the neutral gas is sufficiently low and when we can neglect the contribution of secondary ions, formed in the accelerator, to the concentration, we can write the following system of equations

$$\frac{dj_x}{dx} = \nu_i n_e, \tag{2.1}$$

$$j_x = b_\perp \left\{ n_e \frac{d\varphi}{dx} - \frac{d}{dx}(n_e T_e) \right\}, \tag{2.2}$$

$$\frac{d^2\varphi}{dx^2} = 4\pi e \left\{ n_e - \frac{j_+}{\sqrt{\frac{2e}{M}(\varphi_0 - \varphi)}} \right\}, \tag{2.3}$$ /296

$$T_e = \beta\varphi. \tag{2.4}$$

Here $\nu_i = \langle\sigma_i v_e\rangle n_0$ is the frequency of ionization produced by the electrons; n_0 is the concentration of neutral particles; σ_i is the cross section of ionization produced by electrons; $1 > \beta > 0$, $b_\perp \approx \frac{e}{m\tau\omega^2}$, $\omega\tau \gg 1$.

In (2.1) we have neglected the drift of electrons along the magnetic field. Equation (2.4) is valid when the values of φ_0 are sufficiently large, when we can neglect the average energy loss associated with ionization, which, as we know (ref. 5), does not exceed approximately 100 eV. In this case the ionization frequency ν_i is constant. For the sake of simplicity we shall also assume that $b_\perp$ is constant.

We introduce the dimensionless variables

$$\bar{\varphi} = \frac{\varphi}{\varphi_0}, \quad \xi = \frac{x}{l}, \quad \bar{n}_e = \frac{n_e}{n^*}, \quad \bar{j} = \frac{j_x}{j^*}, \quad \bar{T}_e = \frac{T_e}{\varphi_0},$$

where

$$l = \left(\frac{e\varphi_0}{\nu_i\tau m\omega^2}\right)^{1/2} = \frac{\rho_e^0}{\sqrt{2\nu_i\tau}}, \tag{2.5}$$

$$n^* = \frac{j_+}{\sqrt{\frac{2e}{M}\varphi_0}}, \tag{2.6}$$

$$j^* = \nu_i n^* l. \tag{2.7}$$

In the future we shall drop the bar over the dimensionless variable. Then in place of (2.1)-(2.4) we obtain

$$\frac{dj}{d\xi} = n_e, \tag{2.8}$$

$$j = n_e\frac{d\varphi}{d\xi} - \frac{d}{d\xi}(n_e T_e), \tag{2.9}$$

$$\alpha \frac{d^2\varphi}{d\xi^2} = n_e - \frac{1}{\sqrt{1-\varphi}}, \qquad (2.10)$$

$$T_e = \beta\varphi, \qquad (2.11)$$

where $\alpha = \frac{n_e^0}{n^*} = \frac{j_0}{j_+}$, $n_e^0 = \frac{\nu_i \tau m \omega^2}{4\pi e^2}$ is the concentration of the electrons in the "vacuum" state, when we can neglect the concentration of the ions, $j_0 = n_e^0 \sqrt{\frac{2e}{M}\varphi_0}$ is some characteristic current.

We write the boundary conditions in the form

$$\varphi(0) = \frac{d\varphi}{d\xi}(0) = j(0) = n_e(\xi_0) = 0. \qquad (2.12)$$

The condition

$$\varphi(\xi_0) = 1 \qquad (2.13)$$

determines the length of the acceleration region or the thickness of the layer close to the anode where we have the entire potential drop.

The values $\xi=0$ and $\xi=\xi_0$ fix the beginning of the layer and the position of the anode respectively.

By selecting the boundary conditions in the form (2.12) we thus consider the case of accelerator operation with a cold cathode when the Hall current is generated as result of spatial ionization.

For the sake of simplicity we shall first consider the case $\beta=0$. Then by /297
eliminating j and n_e by means of (2.9) and (2.10), we obtain an equation for φ

$$2\frac{d^2}{d\xi^2}(\sqrt{1-\varphi}) + \frac{1}{\sqrt{1-\varphi}} + \alpha\left\{\varphi'' - \frac{d}{d\xi}(\varphi'\varphi'')\right\} = 0. \qquad (2.14)$$

The primes in (2.14) designate differentiation with respect to ξ.

When $\alpha \to \infty$ we take into account (2.12) and (2.13) and obtain the following for the "vacuum" state.

$$\varphi_{\text{vacuum}} = \frac{\xi^2}{2}, \qquad (2.15)$$

$$\xi_{0\ \text{vacuum}} = \sqrt{2}. \qquad (2.16)$$

Assuming that $\alpha=0$ in the zero approximation, we take into account (2.12) and (2.13) and obtain the following from (2.14):

$$\left.\begin{array}{c}\xi=\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\frac{1}{2}\ln\frac{1}{1-\varphi}}\right),\\ \xi_0=\sqrt{\pi}\end{array}\right\} \qquad (2.17)$$

where

$$\mathrm{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt.$$

Substituting the resulting solution into the term which has been dropped in (2.14) we find that its relative magnitude is of the order of α in the entire range over which ξ and φ vary. As we can see from (2.10) the solution when $\alpha \to 0$ corresponds to a quasi-neutral layer, i.e. the positive space charge of ions is compensated everywhere by the electrons and the accelerated current is not limited by the space charge. More precisely (this is easy to show)

$$|\Delta n_e|=|n_e-n_+|\approx n_e^0 \ll n^*.$$

In this case we have a double layer: $\frac{d\varphi}{d\xi}=0$ when $\varphi=1$. The thickness of the ξ_0 layer compared with the thickness of the layer in the vacuum state $\xi_{0\,\mathrm{vacuum}}$ remains practically unchanged

$$\xi_0=1.25\xi_{0\,\mathrm{vacuum}}.$$

The electron concentration at the anode does not approach zero but rather tends to increase. In the case $\beta=0$ the boundary condition $n_e(\xi_0)=0$ is no longer required because the derivative $\frac{dn_e}{d\xi}$ no longer appears in the system (2.8)-(2.11).

The physical meaning of the fact that the layer thickness does not differ too much from the vacuum case when ions are present, is illustrated by the following simulated problem in which we do not assume quasi-neutrality. Let us assume that the concentration of ions in system (2.8)-(2.11) is equal to $n_+=\gamma n_e$ where $\gamma<1$ and $\beta=0$. Then for the boundary conditions (2.12)-(2.13), excepting the condition $n_e(\xi_0)=0$, we have $n_e=\frac{1}{1-\gamma}n_e^0$, $\varphi=\frac{\xi^2}{2}$ and $\xi_0=\sqrt{2}$. We can see that the presence of ions leads to an increase in the electron concentration by a factor of $\frac{1}{1-\gamma}$ and that $\Delta n_e=n_e-n_+=n_e^0$, i.e. the excess of electron concentration is exactly equal to the "vacuum" concentration.

In this case the layer thickness and the potential distribution remain unchanged since the generation and drift of electrons is proportional to their concentration which remains constant over the layer.

Let us now take into account the electron pressure gradient. In this case /298 instead of (2.14) we have

$$\frac{d^2}{d\xi^2}\left\{2\sqrt{1-\varphi}+\beta\frac{\varphi}{\sqrt{1-\varphi}}\right\}+\frac{1}{\sqrt{1-\varphi}}+ \\ +\alpha\left\{\varphi''-\frac{d}{d\xi}(\varphi'\varphi'')+\beta\varphi^{IV}\right\}=0. \qquad (2.18)$$

We write the expression for the current density when $\alpha \to 0$

$$j=\frac{(1-\beta)-\left(1-\frac{\beta}{2}\right)\varphi}{(1-\varphi)\sqrt{1-\varphi}}\frac{d\varphi}{d\xi}, \qquad (2.19)$$

The coefficient in front of $\frac{d\varphi}{d\xi}$ contained in (2.19) becomes equal to 0 when $\varphi=\varphi^*$

$$\varphi^*=\frac{1-\beta}{1-\frac{\beta}{2}}. \qquad (2.20)$$

On the other hand,

$$j=\int_0^{\xi} n_e d\xi > 0.$$

This means that the first and second derivatives of the potential become equal to infinity when $\varphi \to \varphi^*$. Thus quasi-neutrality is disrupted at this point. The electron pressure gradient increases so rapidly that the flux of electrons produced by the electric field is compensated by the inverse flux produced by the pressure gradient and the electron concentration increases sharply.

We shall first consider the solution of (2.18) in the quasi-neutral region $0 \leq \varphi < \varphi^*$.

By introducing the new variable

$$y=\frac{1}{2}\left(\frac{2-\beta}{n}+\beta n\right), \qquad (2.21)$$

where

$$n=\frac{1}{\sqrt{1-\varphi}}, \tag{2.22}$$

equation (2.18) may be rewritten in the following manner when α=0:

$$2y''+\frac{1}{\beta}\{y-\sqrt{y^2-\beta(2-\beta)}\}=0. \tag{2.23}$$

The boundary conditions are as follows:

$$y(0)=1, \tag{2.24}$$

$$y'(0)=0. \tag{2.25}$$

The solution of (2.23)-(2.25) has the form

$$\xi=\sqrt{2\beta}\int_y^1\left\{1-S^2+S_*^2\left(\operatorname{arch}\frac{1}{S_*}-\operatorname{arch}\frac{S}{S_*}\right)-\right.$$
$$\left.-(\sqrt{1-S_*^2}-S\sqrt{S^2-S_*^2})\right\}^{-\frac{1}{2}}dS, \tag{2.26}$$

where $S_*=\sqrt{\beta(2-\beta)}$.

The figure shows the potential distribution for different values of β. The thickness of the quasi-neutral region ξ^* when β=0.5 is $\xi^*\approx0.7$ and φ^*=0.67. /299

It is necessary to take into consideration the condition $n_e(\xi_0)=0$ in the region $\varphi\to1$. Here we have another limiting case when we can neglect the electron concentration: the motion of electrons ceases to have a diffusion nature and there is a breakaway of electrons to the anode. The potential distribution in this region is determined by the ion space charge.

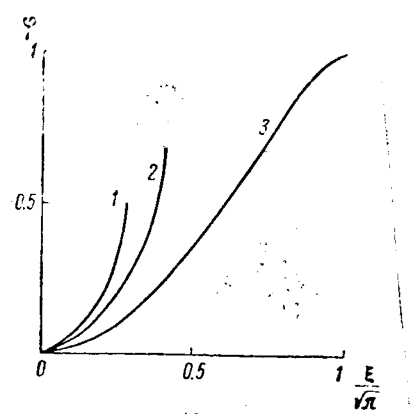

Figure. Potential distribution in the quasi-neutral region for different values of β. 1) β=2/3; 2) β=0.5; 3) β=0

Thus when $\varphi\to1$ we have the following, in place of (2.10):

$$\alpha\varphi'' = -\frac{1}{\sqrt{1-\varphi}}, \tag{2.27}$$

which gives us

$$\xi_0 - \xi = \frac{2}{3}\sqrt{\alpha}(1-\varphi)^{3/4}. \tag{2.28}$$

The size of the breakaway region is determined by the Larmor radius of the "heated" electron and the accelerated ion current, in this manner, is limited by the space charge.

Obviously the limiting current in this case is proportional to the square of the magnetic field

$$j_+ \sim H^2\sqrt{\varphi_0}. \tag{2.29}$$

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